# Draft Implementation Plan: Changes in HydroDyn to Support Time-Varying Buoyancy Loads on Morison Members

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# General Notes

This plan has all hydrodynamic loads be calculated on each node of a Morison member as a lumped rather than distributed load. There are no more distributed loads. The computation process should be done member-by-member, with each member adding forces to the applicable nodes. Joint nodes attached to multiple members will then see forces from each member.

Members are no longer split at boundaries (i.e. at the waterplane, the water ballast level, or the seabed). Instead, the transitions are handled across an element and can change location as the member moves. This reduces the discretization requirements.

Hydrostatic-type forces (e.g. buoyancy and weight) are now calculated based on instantaneous position and orientation. Since some forces are now constant and others are time-varying, the constant versus time-varying quantities need to be clearly distinguished. Related to that, both reference and instantaneous orientation transformations will be needed depending on the load in question.

## Changes to the HydroDyn input file:

* None

## Module-level input and output changes:

* Remove distributed input and output meshes
* Keep existing lumped input and output meshes to contain all nodes of the Morison members
	+ First entries are the joints
	+ Following entries are the *interior* nodes for each member in turn
	+ There is no duplication of nodes at joints

## General Code Changes

* Change “Morison\_MemberType” to represent members rather than individual elements.
* Rename the Morison\_MemberType array from “elements” to “members” and use it to hold the member objects.

## Misc. Code Comments

* I think there was a sign error in DistrAddedMass (subtracting rather than adding f2 term)
* I think there was an indexing typo in the calculation of m%D\_F\_AM\_F in Morison\_CalcOutput (maybe this could be responsible for the issue described in <https://github.com/OpenFAST/openfast/issues/245>)

# General Morison Member Treatment

To simplify the calculations, hydrostatic loads are derived based on a two-dimensional representation and then converted back into three dimensional components afterward.

A Morison member is defined as connecting two joints, with global coordinates for the starting joint and for the ending joint. The start point is defined as the end point that has a lower coordinate value. If the two end points have the same Z coordinate value, then the one that has the lower coordinate value is the start point. If the two end points have the same and coordinate value, then the one that has the lower coordinate value is the start point.

The member is divided uniformly into elements of length . There are then nodes. Loads on an element are formulated with respect to node and are then spread by factor alpha to node .



The member’s starting and ending radii are denoted and , respectively, and the vertical elevations of these nodes are and , respectively.

Transformation matrix is already used to convert from member coordinates to global coordinates and is the inverse/transpose of the direction cosines matrix used in OpenFAST:

where L denotes member length (with subscripts indicating projected length in the given plane):

The third column of is the unit vector indicating the direction of the member axis from start to end joint. This vector is denoted and is used extensively. or without a subscript indicates the instantaneous orientation of a member as a whole based on its end point coordinates, while or denotes the member reference (undisplaced) orientation.

For element-level effects, or refer to the instantaneous orientation of element calculated based on the instantaneous displacements of its two nodes, and , (ignoring the orientation input from the mesh).

For end effects, use the orientation of each element that connects to the joint.

The variable indicates the coordinate along the length of the member in a relative sense, and r indicates the radius at a given location along the member. The member’s inclination angle from vertical is denoted and the heading or the incline from the X direction around the Z axis is . Similarly to the nomenclature for or , and denote the reference angles for the member, and and denote the instantaneous angles for a given element based on its node positions.

If **C** is the direction cosines matrix for the Morison member, then the incline angle (always positive, that is, ) is

and the heading () is

Element-level values are calculated equivalently. Note that for surface- or seabed-piercing members the incline angle is effectively constrained to be less than horizontal () by various limitations enforced in each situation. For fully submerged members, the incline angle will never become much more than 90° because of small platform rotations.

## Water line offsets, wave stretching, and changing element submergence

The distribution of hydrostatic and hydrodynamic loads changes when an element is not fully submerged.

Hydrostatic calculations are done in the absence of waves and define an element’s state of submergence based on whether each of its nodes are above or below the still water line, which is at the Z location specified by MSL2SWL. In practice, the water depth and Z coordinates in HydroDyn are adjusted by MSL2SWL so that Z=0 corresponds to the still water level. This means that WtrDpth = WtrDpth(input) + MSL2SWL and the node Z coordinates in the Morison module should be offset from the input mesh:

A similar offset needs to be applied to any other z coordinate quantities that are intended to remain consistent with the structure z coordinates, such as flooded ballast fill levels and marine growth depths.

An element’s state of submergence and lumping of loads on adjacent nodes is set as follows:

|  |  |  |
| --- | --- | --- |
|  |  | Not submerged – no hydrodynamic loads |
|  |  | Partially submerged – loads shared between nodes and  |
|  |  | Fully submerged – loads shared between nodes and  |

To avoid an abrupt change in forces and moments when a node crosses the waterline, smoothing techniques are used and loads on a partially submerged element are lumped at the two closest nodes below the waterline.

Hydrodynamic load calculations account for the instantaneous free surface elevation including the effect of waves. This is explained in the hydrodynamic loads section.

## Treatment of loads at or below the seabed

HydroDyn is intended to not model any loads on buried parts of members. As an approximation to this, external buoyancy and internal buoyancy (water ballast weight) loads on member sides are neglected for any elements fully below the seabed but are included for the entirety of the seabed-piercing element. The other loads on member sides, listed below, are neglected for any fully buried element *and also for the buried portion of the seabed-piercing element*:

* Marine growth weight and inertia
* Water ballast inertia
* Hydrodynamic loads: drag, added mass, hydrodynamic inertia force

All of the loads calculated for member ends are neglected when the end node is below the seabed. Member end loads are calculated for nodes at and above the seabed.

The seabed boundary is only checked in the reference (undisplaced) configuration and is not checked again in time.

## Treatment of member flexibility

Member flexibility is handled by using instantaneous values for node coordinates when computing certain forces. For instance, the instantaneous orientation of element is taken from the coordinates of nodes and passed in from the glue code (from a structural module). End effects, which are only added at joints, use the instantaneous orientations of the connected elements.

At present, only hydrostatic-type forces are based on the instantaneous member pose; the other loads are based on the reference pose. Accordingly, orientation changes due to member flexibility are only relevant for hydrostatic-type loads at this time.

## Standard functions for tapered or non-tapered cylindrical elements.

Moments of inertia are relative to the start node of the element. A negative value of indicates measurements are relative to the end (i+1) rather than start (i) node (this is reflected in the sign of below).

**Non-tapered case** ():

Radial moment of inertia about node 1:

Axial moment of inertia:

**Tapered case** (note: ):

Radial moment of inertia about tip of cone:

Radial moment of inertia about node 1:

Axial moment of inertia:

## General pre-computed quantities (using reference values)

For seabed-piercing members (; ):

where is the reference Z elevation of the lowest node above the seabed. (Note: this equation is invalid at ; see limitation below)

Marine growth is specified in the input file in terms of thicknesses at different depth points, with interpolation between these points. To be consistent with node z coordinates, the marine growth depths read in from the input file must be shifted by -MSL2SWL before they are used for determining marine growth thicknesses at the Morison member nodes.

The effect of marine growth is applied to the nodes along each Morison member, with each node assigned a marine growth thickness and marine growth density at initialization. These values are interpolated from the marine growth inputs based on each node’s depth at in the reference (undisplaced) configuration.

For brevity in later equations, the variable is used to represent the radius at node inclusive of marine growth:

The uniform taper of the member before counting the effect of marine growth is denoted (without a subscript).

Because of the possibility of non-uniform marine growth, the taper with marine growth included needs to be calculated separately at each member element

The taper of the internal surface of a member is needed for flooded ballast calculations and is denoted

Several calculations require knowing the relative position of an element’s volumetric centroid between its two nodes. For a fully submerged cylindrical element that may be tapered and may have marine growth, the relative position of the centroid between node and node is

The quantities and should be calculated for each element in the member at initialization in the reference (undisplaced) configuration. = 0 corresponds to the centroid being at node and = 1 corresponds to the centroid being at node .

The quantity for any element containing water ballast:

## Misc. Limitations

* Near-horizontal members cannot cross the seabed boundary (but they can sit on it) because this would create modeling inaccuracies. However, this situation is avoided by a general constrain on the incline of seabed-piercing members.

## Output of Distributed Loads

Distributed loads along a member can be output at each node point following the following approach

where is the lumped load in question at the node and is the length of the member’s segments.

This applies to all the of the loads discussed in the following sections except for end loads. End loads can only be output in lumped form at joints. The same approach for specifying joint and member outputs in the HydroDyn input file is retained.

# External Buoyancy Forces

## Sides - Fully Submerged Elements

The buoyancy loads on fully submerged elements are calculated based on the solution of a pressure integration around the side wall of the element. This pressure integration considers the instantaneous wetted area of the member, assuming still water (zero wave elevation).

**Below the seabed**, buoyancy forces are neglected on all fully buried elements and on the member end if its node is below the seabed. Full buoyancy loads are included on the sides of the seabed-piercing element—these are expected to be of negligible consequence as long as there are no highly tapered or angled seabed-piercing members (otherwise an error should be thrown – see the limitations section below).

### Calculated at every time step

We start by computing the net axial force, transverse force, and moment (about node ):

Once the force components and moment are calculated for the element, they need to be distributed between its two nodes. This distribution is done based on the relative location of the element’s center of buoyancy (submerged centroid) between the two nodes and based on the nodes’ water depths. The forces and moments will then be distributed to the nodes according to and , where is any force or moment mentioned above and (note: and )

(Note the cube exponent in the above equation may change to 2 or 1 depending on further analysis of smoothing effects.)

Before distributing the moment, it must be adjusted to avoid double counting the moment caused by distributing the radial force component. The force distribution will account for all moments except for those originating from the unbalanced longitudinal pressure components due to taper. Accordingly, for a submerged element the remaining moment will only be nonzero when the element is tapered.

Lastly, the heading and inclination angle must be applied to convert the loads into the global coordinate system:

These are the buoyancy loads from element that are to be *added* to nodes and node (these nodes will likely have additional loads on them as well).

## Sides - Partially Submerged Elements

For partially-submerged elements (those that cross the waterplane), the buoyancy load is calculated based on the displaced volume of that element. For the purposes of this calculation, a variable is used to represent the distance along the element axis from the submerged node to the waterplane. Tapered and non-tapered members are handled differently.

### Calculated at every time step – Tapered Case

For a tapered element, the calculations require knowing If the cone’s radius at the point its centerline intersects the waterplane is ,

equations for the submerged volume and centroid of an inclined tapered cylinder are

where

These equations hold whether the taper is positive or negative, and also provide the desired effect when the definition of the submerged volume includes a negative-volume region (i.e. when the plate at node is only partially submerged). However, they are not defined for a non-tapered cylinder, so that condition must be detected (when the absolute value of is below some threshold, say 0.001) and then handled with simpler calculations.

### Calculated at every time step – Non-Tapered Case

When there is no taper, the calculations are based on a truncated cylinder geometry. The submerged volume is

### Calculated at every time step – All Cases

The horizontal center of buoyancy location relative to node is

The buoyancy force and moment can be easily calculated based on the displaced volume and centroid. However, because the element is part of a larger member, only the portion of load resulting from the pressure around the side wall of the element should be used. The contribution from the bottom plate that would be exposed to the water if the element were in isolation must be subtracted (this is done in the second terms in and below). The total displacement-based forces and moments from element acting about node are

These loads need to be distributed to nodes in a continuous way, even as elements transition in and out of the water. This is done by modifying the original centroid-based distribution according to the nodes’ elevations, such that the force or moment goes to zero when a node is at or above the waterplane. Furthermore, to avoid putting any buoyancy load on nodes above the waterline, the loads from a partially submerged element are distributed to the element’s lower node, and to the node below that one. The modified distribution factor is calculated as (note: and )

and the forces and moments from element are distributed to nodes and according to

As with the fully submerged case, the moment needs to be adjusted to account for the distribution of the radial force down the member. Because the forces are shifted down an element length, the corrected moment is

The last step is to convert back into global coordinates based on the heading of the member’s incline.

## Ends

Calculation of loads on member ends is unchanged from the previous implementation except that hydrostatic loads should be updated at each time step based on instantaneous depths and orientations. When the ends of multiple members intersect, the end node is common among the members and will take the summation of the end forces and moments from each member.

Buoyancy loads are neglected for any end whose node reference depth is below the seabed or instantaneous height is above the still water line.

### Calculated at every time step

The **buoyancy load on each end** of a submerged member is calculated as follows, where the angles are the instantaneous values of the attached elements:

For node 1:

Or for node if it is submerged:

where, for either end, the axial force and moment terms are

## Limitations

* Because the buoyancy of a partially submerged element is added to the buoyancy of the adjacent fully submerged element of the same member, HydroDyn will not allow any element attached to a joint below the SWL to be partially submerged. Therefore, the first/bottom-most element of a submerged member must always be submerged.
	+ An error should be triggered if at any point while .
* The upper end plate (at node ) must not cross the water plane. This prevents near-horizontal members crossing the free surface.
	+ An error should be triggered if
* The lower end plate () must not cross the water plane. This prevents near-horizontal members crossing the free surface.
	+ An error should be triggered if
* Because model limitations may become problematic for highly tapered or angled members crossing the seabed, this should be prohibited for the time being.
	+ An error should be thrown at initialization if or abs(m) > 0.1 while the reference and .

Note: Increasing the element resolution of members crossing the SWL will improve the buoyancy load distribution of the solution.

# Marine Growth Weight and Inertia Force

Weight and inertia from marine growth along the member sides is calculated on an element-by element basis. The effect of taper on the marine growth mass and distribution about a single node is included. Weight and inertia form marine growth on the member ends is calculated on a per-joint basis. Marine growth depths are adjusted by -MSL2SWL in the process of calculating marine growth thicknesses at each node during initialization.

**Below the seabed**, marine growth loads are neglected.

## Sides

Each element is split into halves, and the loads from each half added to the adjacent node. If the member passes below the seabed, only the portion of the seabed-piercing element above the seabed is considered for loads.

### Pre-computed at initialization

The following are computed using standard functions for each of the two half-elements:

* Vinner, half-element displaced volume before marine growth
* Vouter, half-element displaced volume including marine growth
* hc,inner – distance from respective node to center of buoyancy before marine growth
* hc,outer – distance from respective node to center of buoyancy including marine growth
* I\_linner, Ir\_inner – axial and radial moments of inertia of solid from member radius
* I\_louter, Ir\_outer – axial and radial moments of inertia of solid from marine growth surface

Mass of each half segment’s marine growth.

where is marine growth density.

Center of mass of each half-segment’s marine growth:

Radial and axial moments of inertia about the respective node of each half segment:

### Calculated every time step

The calculation for marine growth weight and inertia loads for each two half-elements are calculated similarly.

First ,the moments of inertia matrix in global orientation for the upper and lower half-segments of each segment about their respective center of masses are:

where the plus and minus subscripts denote values for the upper and lower half-segments, respectively.

The loads on node from the lower half-element are

where and are calculated using the values for the lower half-element, is the lower node translational acceleration, is the lower node rotational acceleration, and is the lower node angular velocity vector.

The loads on node from the upper half-element are (where is now negative)

where and are calculated using the values for the upper half-element. The same approach is used for the unburied portion of an element that crosses the seabed, where the length used is (and the term is zero).

## Ends

Marine growth is calculated on a per-joint basis, meaning the calculation accounts for all intersecting member ends. This avoids unrealistic amounts of marine growth at joints between members.

Marine growth loads are neglected for any joint whose reference depth is below the seabed.

### Pre-computed at initialization

Marine growth mass on member ends intersection at a joint can be calculated by applying the marine growth thickness to the weighted normal area vector magnitude () for the joint:

where or and

Marine growth radial and axial moments of inertia are calculated at a joint as follows:

with the weighted normal reference moment of inertia vector

where is the outward-facing normal vector of the end in question in the reference configuration:

In the above, m represents a member attached to the joint and M represents the total number of members attached to that joint.

Lastly, the inertia matrix needs to be calculated. If the weighted normal reference moment of inertia vector is zero (), then the inertia matrix should be set to zero. Otherwise, it is calculated as:

where is a rotation matrix, calculated by an application of Rodrigues’s rotation formula for a 180° rotation about an axis half way between the Z axis and the direction of as follows::

where

### Calculated every time step

The marine growth weight on the joint is:

The marine growth inertia force and moment vector on the joint is

## Limitations

Marine growth thicknesses are calculated about the node points of the discretized member and distributed linearly along the member axis between node points. Marine growth is assumed to stop abruptly at the point where the member axis passes below the seabed, if applicable. In all cases, the marine growth is modeled as axisymmetric, meaning that there is potential for deviation from the marine growth being strictly a function of depth.

# Internal Buoyancy (Water Ballast Weight)

Weight of internal water ballast is modeled in a similar way to the buoyancy loads, except that the water ballast free surface is assumed to be perpendicular to the member axis at all times. Quantities are therefore calculated assuming no shifting of the ballast water. Although members might be compartmentalized, this formulation assumes that the ballast could be pressurized, for example to balance external water pressure, which would result in equivalent net axial loads on internal nodes as an uncompartmentalized case. Members are assumed to be sealed, so that no fluid can enter or exit any member.

If the member is partially filled, i.e., (note: in this case, , the fill level along the axis should be calculated in the reference configuration and saved at initialization:

where the fill level, has been adjusted by -MSL2SWL to remain with the node Z coordinates (which have all been adjusted by -MSL2SWL). Note that although this filled length is held constant, in future it could be made variable to model active ballast. The coordinate at each node is also used in the calculations: .

If the member is fully or over filled ( in the reference configuration at initialization), is not used. Instead, the height to the fill level is stored (note: :

This value, which is set at initialization and remains constant, represents the pressure of the water ballast at the instantaneous upper end of the member (i.e. if a small pocket of air was trapped in the member, would be its pressure). Neglecting member deformation, this pressure remains constant and is added to whatever hydrostatic pressure profile exists due to the elevation change over the length of the member axis (radial pressure variations are neglected). For example, if the fill level was set to the still water line, then the ballast pressure would be equal to the outside water pressure, which might represent a flooded monopile. For this pressure to remain constant, this assumes that each member is completely compartmentalized from other members.

**Below the seabed**, water ballast weight loads are neglected on all fully buried elements and on the member end if its node is below the seabed. Loads due to water ballast pressure on the full length of the seabed-piercing element are included—these are expected to be of negligible consequence as long as there are no highly tapered or angled surface-piercing members.

## Sides – fully flooded elements

### Pre-computed at initialization

Reference depth-based force distribution adjustment for each submerged element:

The following constants are computed only once for each element and related to axial load due to side pressure, radial load due to tilt, and moment due to tilt:

### Calculated at every time step

The axial force, radial force, and moment of element concentrated at node are:

where is set to zero when a member is not fully flooded and where

The moment is adjusted to account for the redistribution of radial load using:

Distribution of forces and moments to lower and upper nodes of the element:

## Sides – partially filled element (the element containing the upper limit of the ballast )

This is the element such that in the reference configuration at initialization.

The flooded ballast’s upper surface is assumed to be perpendicular to the member axis, approximating a compartmentalized condition for larger member diameters rather than allowing water to flow from one side of the member to the other.

### Pre-computed at initialization

The load distribution for the partially flooded element depends on its nodes’ reference depths:

(note that and )

The fill level in the element is

The following constants are computed only once for each element and related to axial load due to side pressure, radial load due to tilt, and moment due to tilt:

The last term in the equation for represents the moment resulting from the ballast’s upper end when it is not horizontal, since a compartmentalization is assumed and therefore a plate within the member will be holding the upper surface of the ballast perpendicular to the member axis when the member is inclined.

### Calculated at every time step

The axial force, radial force, and moment of element concentrated at node are:

The moment is adjusted to account for the redistribution of radial load using:

Distribution of forces and moments to element’s lower node and one node *below* that:

## Ends

Ballast weight is neglected for any end whose node reference depth is below the seabed.

### Calculated at every time step

The effect of flooded ballast on an end node is calculated as

where is the axial force magnitude and is the moment magnitude.

**If the member is partially flooded**, only the lower node () has an end load, calculated as

**If the member is fully flooded**, in which case either end may be higher, the end node loads are as follows:

This approach to the end forces gives an approximation for the internal hydrostatic pressure of the ballast water as the member orientation changes.

## Limitations

The following limits are applicable only to partially flooded members ():

* The way loads are lumped from the partially flooded element requires that the first/bottom-most element of a member be fully flooded.
	+ An error should be triggered if in the reference configuration at *initialization*.
* The model will not be accurate if a partially flooded member is close to horizontal or inverted because the formulation assumes the fill free surface spans the member’s circular cross section.
	+ A warning should be triggered if in the reference configuration at *initialization of a partially flooded member*. (This condition is that the true free surface would be touching the end plate, and is roughly proportional to the size of the potential inaccuracy.)

Inaccuracies may occur for tapered fully-flooded members that undergo depth or orientation changes because axial loads along the member do not account for changes in internal pressure.

The water ballast distribution is assumed to by axisymmetric along the member, resulting in non-horizontal boundaries at the ballast free surface and at the seabed intersection (if applicable).

# Inertia Force from Water Ballast

For the purpose of inertia force calculations, flooded ballast is assumed to move entirely with the member, as if the member had compartments and baffles, which is likely in most cases. The most important exception to this would be a flooded monopile, which would not normally experience internal water inertial forces from yaw or heave motions. However, because the yaw and heave deflections of a monopile are very small, this model discrepancy is likely of negligible importance.

The force required to accelerate the flooded ballast is modeled similarly to the marine growth inertia force. Each fully flooded element is divided into two half-elements, and the inertia load from each half-element’s ballast is lumped to the respective node. In addition, the flooded portion of a partially flooded element is modeled in the same way as a half-element, except with the length adjusted to be the flooded length. Within this discretization, the inertial forces are calculated exactly.

**Below the seabed**, water ballast inertia is neglected (only the exposed portion of the seabed-piercing member is considered for ballast inertia loads).

## Sides

### Pre-computed at initialization

The following ballast-specific quantities are computed using standard functions for each of the two half-elements, as well as for the flooded portion of the partially flooded element if applicable:

* V – flooded volume of half-element or flooded portion of partially flooded element
* – distance from respective node to flooded center of mass
* – axial and radial moments of inertia of solid from member radius

### Calculated every time step

First, the moments of inertia matrix in global orientation for the upper and lower half-segments of each segment about their respective center of masses are:

where the plus and minus subscripts denote values for the upper and lower half-segments, respectively.

The inertia loads for each two half-elements are calculated similarly. The loads on node from the lower half element are

where and are calculated using the values for the lower half-element, is the lower node translational acceleration, is the lower node rotational acceleration, and is the lower node angular velocity vector. The same approach is used for the inertial load of the partially-flooded element’s filled portion, except the length used in the calculations is rather than (and the i+1 term is zero).

The loads on node from the upper half element are (where is now negative)

where and are calculated using the values for the upper half-element. The same approach is used for the unburied portion of an element that crosses the seabed, where the length used is (and the i term is zero).

## Ends

No separate end inertia is included.

## Limitations

The water ballast distribution is assumed to by axisymmetric along the member, resulting in non-horizontal boundaries at the ballast free surface and at the seabed intersection (if applicable).

# External Hydrodynamic Loads

Hydrodynamic loads (drag, added mass, inertia) are to be calculated based on the reference (undisplaced) position and orientation of the member. When the member unit vector is used, it refers to the reference state of the member and should be set at initialization.

## Sides

Because these loads are now lumped, a term has been added to the previous equations. Also, the end nodes see half the load of the interior nodes, as well as an additional moment, because they lack the symmetry of two adjacent half-elements with which the interior nodes are modeled. As is done in the existing implementation, these loads can simply be switched off for nodes that are not submerged. If the member crosses the waterline, the node immediately below the waterline accounts for all the hydrodynamic loads above it, and its forces and moments are adjusted accordingly.

The taper term in the previous implementation, abs(dRdZ), is replaced with a node-specific value that includes the effect of marine growth for the adjacent elements on the node’s calculations. The taper term is equal to for node 1, for node N+1, and for each interior node .

**Inclusion of wave stretching** has the effect of changing the free surface elevation defining what is submerged versus not submerged, and the magnitude of wave velocities and accelerations at each node. The water surface elevation, velocity, and acceleration at each node’s reference (undisplaced) location is precalculated, thus no change is needed in the calculations that incorporate these values. The only change needed is incorporation of the surface elevation at each member’s initial free surface crossing location, denoted .

Because members are no longer being split at the mean water line, an additional node should be created in HydroDyn where the member axis crosses the waterplane, i.e., , strictly for the purpose of providing wave elevation data. The horizontal coordinates of this point are set in the reference configuration at initialization as

Because the buoyancy calculations do not support near-horizontal members crossing the water plane, the above equation is always defined.

In this implementation plan, represents the free surface elevation when wave stretching is enabled, and zero when wave stretching is disabled.

### General Form of Hydrodynamic Forces and Moments

Hydrodynamic loads can be calculated by looping through the nodes along the member in order, stopping upon encountering a node whose reference position is above the waterline. Hydrodynamic loads lumped to each node take the following pattern, where represents a drag, added mass, or inertial force-per-unit-length vector at node . represents the resulting size-6 force-and-moment vector to be applied on the node.

**Quantities used in hydrodynamic force calculations**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | End node 1 (if not in seabed) | Node just above seabed | Fully submerged interior node | Node just below free surface | End node N+1 (if submerged) |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Note that although reference positions are used for the node depths, the wave elevation term is time-varying when wave stretching is enabled, meaning that the value of and the index of the node just below the free surface must be computed at each time step.

The free surface location along the axis from the topmost submerged node is

and the seabed location along the axis from the bottom-most un-buried node is

Note that horizontal members crossing the seabed boundary, just like crossing the free surface, are not permitted (a limit has been added for this).Following are the calculations for each type of hydrodynamic force-per-unit-length vector that can be inserted into the node force-and-moment equations above. These are similar to the existing implementation, but there are some modifications and a number of additions. Many use a common matrix that describes the member orientation, defined as

This matrix, and the matrix should only be calculated once at initialization based on the reference orientation.

### Hydrodynamic Drag Force

The drag force-per-unit-length is being updated to include an axial term. The axial drag coefficients must be added as new axial coefficient columns (for with and without marine growth) in the input file.

where is the instantaneous water velocity at the node’s initial location relative to the node’s instantaneous velocity, and

### Hydrodynamic Added Mass Force

On an individual element level, the added mass implementation accounts only for translational accelerations and does not include reactions to rotations. The three-by-three translational added mass-per-unit-length matrix for a given node, including both transverse and axial components, can be calculated as

The added mass force-per-unit-length vector is

where denotes the node acceleration.

### Hydrodynamic Inertia Force

where is the fluid acceleration at the initial node position and

## Ends

Calculation of hydrodynamic loads on member ends is unchanged from the previous implementation. When the ends of multiple members intersect, the end node is common among the members and will take the summation of the end forces and moments from each member. When the end node is at or above the seabed, all loads are calculated. When the end node is below the seabed, hydrodynamic loads should not be calculated.

### Hydrodynamic Drag Force

Calculation of drag on the member end(s) is unchanged from the previous implementation:

with the weighted normal reference area vector

where is the outward-facing normal vector of the end in question:

In the above, m represents a member attached to the joint and M represents the total number of members attached to that joint.

### Hydrodynamic Added Mass Force

Calculation of added mass force on the member end(s) is unchanged from the previous implementation.

with weighted normal volume vector

### Hydrodynamic Inertia Force

Similarly, the external inertia force is

where is the fluid acceleration at the node.

# Derivation Notes for Buoyancy Loads on Member Sides

For a tapered member, the derivation imagines that the element is part of a cone, from which calculations can be done more easily.

 

In the case of a positively-tapered element, the cone end is located a distance down along the element axis, where . The cone from the plate at node downward then has a volume of and its centroid is located ¾ of the way up from the cone’s end, at a position along the element axis.

If the cone is extended upward until the waterplane, the volume is more difficult to calculate. Based on the cone’s radius at the point its centerline intersects the waterplane, the distance from the cone’s end to the waterplane-axis intersection is . The depth of the cone end is

The waterplane area of an inclined cone has the shape of an ellipse, and the ellipse’s center is offset from the cone axis. With some work, the ellipse’s dimensions and offset from the cone axis can be identified as

These constants are defined such that the equation for the ellipse formed by taking a horizontal slice of the inclined cone at any elevation, , from the cone end point is

where and are local horizontal coordinates relative to the member axis at that elevation.

Integrating the area of these ellipses vertically over the extent of the submerged cone gives a volume of and a centroid located ¾ of the way up from the cone and at an x offset from the cone end of

To know the buoyancy on the element in question, we need to know the volume and centroid of this partially submerged element in isolation (shaded in Figure X). This is calculated by the difference of the slanted elliptical cone going through the waterplane, and the cone going from the disc at node , as illustrated below.

