## Implementation Plan: Modifications to State-Space Modules in HydroDyn to Support Multiple WAMIT Bodies

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The descriptions that follow cover changes required in the SS\_Radiation and SS\_Excitation HydroDyn modules. It is assumed that the state-space model matrices match the associated WAMIT data and follow NBodyMod 1/2/3 conventions in terms of forces and moments being about the local body coordinate system and wave kinematics being about the global WAMIT reference point (which in the case of NBodyMod = 2 is shifted to each body’s initial position). The required changes in the state-space preprocessing are left for later work.

# SS\_Radiation

### Background

The state-space radiation approach replaces the convolution integral from the Cummins equation with a state-space approximation:

Vector is the platform velocities and vector is the resulting radiation force/moment vector (size 6 if all platform DOFs are enabled). Vector contains the states used to model the radiation for the platform DOFs enabled. Matrices , , and are produced by the preprocessor SS\_Fitting based on radiation data from WAMIT.

The preprocessor treats the DOFs independently, meaning that most entries in the matrices are zero. The matrix entries are arranged in a custom way that deviates from the natural order of the indices, so the pattern of nonzero entries is not simple. Because SS\_Radiation uses single matrices for all DOFs without any expectation of sparseness, HydroDyn does not need to know the order that the matrices or the state vector are arranged in. Speed improvements could be realized by splitting the state-space model and doing separate computations for each DOF of platform velocity, reducing the size of the matrix multiplications, but the significance of these improvements to overall simulation time might be negligible.

The core operation in HydroDyn for state-space radiation forces is simply computing the two matrix equations above.

### Modifications for Multiple Bodies and Different NBodyMod Cases

Expansion of the state-space model to handle multiple bodies follows a similar approach to that of the convolution-based radiation model. The matrix sizes required are as follows, assuming each body has all 6 DOFs enabled. Variable denotes the total number of states used in each model. If NBodyMod = 1, there is a single model with states for all of the bodies in combination. If NBodyMod = 2 or 3, there is a separate model for each body, each with states (the number may be different for each body).

**Original and Modified Array Sizes**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Single Body | NBodyMod=1 | NBodyMod = 2 or 3 |
|  |  |  | , for each body |
|  |  |  | , for each body |
|  |  |  | , for each body |

**For NBodyMod=1**, the form of the state space calculations is unchanged except that the vector and matrix sizes must be expanded to support the new number of degrees of freedom (and resulting increase in number of states). If all platform DOFs are enabled, the input (platform velocity) and output (radiation force/moment) vectors increase to size 6\*NBody, e.g.

where each body’s coefficients are

and is short for the number of bodies, NBody. The change for is similar.

**For NBodyMod = 2 or 3**, the size of the original matrix computations is not changed. Instead, a separate set of computations is done for each body.

**In all NBodyMod cases**, when a body’s heading (PtfmRefztRot) is nonzero, a correction must be made to transform the matrices from local to global coordinates. This could be done in one of two ways:

1. Transforming body velocities from global to local orientations, and converting resulting radiation forces from local to global orientations, at each time step.
2. Transforming the state space model matrices during initialization.

Both options are identical in the end and the second option is more efficient so that is what’s detailed. is the rotation matrix for the PtfmRefztRot values of body

The required matrix transformations are as follows.

For the matrices of each body, , *separately* **when NBodyMod = 2 or 3**:

For the global matrices of all bodies *together* **when NBodyMod = 1**:

Because these matrices are diagonal clusters of 3-by-3 submatrices, the multiplications could be done in a piecewise fashion if that was better for implementation. These matrix transformations are similar to those done for the frequency-domain radiation coefficient matrices.

# SS\_Excitation

### Background

The state-space wave excitation approach follows the same general form as the radiation approach except the input—time-shifted wave elevation—is scalar. This makes the input matrix be a vector of the same size as the state vector.

### Modifications for Multiple Bodies and Different NBodyMod Cases

Similarly to SS\_Radiation, when NBodyMod=1, the size of the vectors and matrices is increased, while for NBodyMod=2 or 3, the sizes are unchanged but the data and equations are run separately for each body in turn.

#### Wave Heading

Because the state-space wave excitation data is currently set for a single direction, the wave heading specified in the state-space input file for a body must match the relative wave heading seen by that body in HydroDyn, including a modulo (mod) operation to account for degree wrapping.

**For NBodyMod=1** there is only one input file so the requirement is

**For NBodyMod=2** there are input files (potentially the same one) for each body and the bodies may be rotated within HydroDyn. The requirement for each body is

**For NBodyMod=3** there is a unique input file for each body, and body headings in HydroDyn simply match those in the input data rather than applying an additional rotation, so the requirement for each body is

Because waves from only a single direction are considered, the only rotation that needs to be applied is on the excitation force output of the model, by adjusting the matrix. The transformation to account for body heading offsets (nonzero PtfmRefztRot) is as follows.

For the matrices of each body, , *separately* **when NBodyMod = 2 or 3**:

For the global matrix of all bodies *together* **when** **NBodyMod = 1**:

#### Wave Phasing when NBodyMod=2

Because NBodyMod=2 sees a body offset applied in HydroDyn, an adjustment in the phasing of wave excitation is needed in this case. Without a means of adjusting the state-space coefficients, which are generated by a separate program, the only option is to adjust the wave elevation signal fed to the state-space model for each body. This can be achieved by producing a separate wave elevation time series at the initial position of each body.

where is the discrete Fourier transform of wave elevations at the global origin and

For each body, the computation of the input needs to be adjusted from using to using :

where is the wave elevation computed at the position of the body from the WAMIT mesh at initialization.